Série 1

Exercice I : Calculer les dérivées partielles premières $\frac{\partial f}{\partial x}(x,y)$, $\frac{\partial f}{\partial y}(x,y)$ des fonctions suivantes :

1- $f(x,y) = x^2 + xy + y^4 + 3$
2- $f(x,y) = xe^y + x^2y$
3- $f(x,y) = x^3 + y^3 - 3xy$

Exercice II : Calculer les dérivées partielles premières et secondes $\frac{\partial f}{\partial x}(x,y)$, $\frac{\partial f}{\partial y}(x, y)$, $\frac{\partial^2 f}{\partial^2 x}(x,y)$, $\frac{\partial^2 f}{\partial^2 y}(x,y)$, $\frac{\partial^2 f}{\partial x\, \partial y}(x,y)$, $\frac{\partial^2 f}{\partial y\, \partial x}(x,y)$, des fonctions suivantes :

1- $f(x,y) = 3x^2y - x y^3 - x - y$
2- $f(x,y) = \sqrt{x^2 + y^2}$
3- $x \ln y + y \ln x$
4- $f(x,y) = e^{2x^2+xy+7x+y^2}$
5- $f(x,y) = \sin xy$

Exercice III : Calculer la différentielle des fonctions suivantes :

1- $f(x,y) = \frac{x^2+xy}{y^2}$
2- $f(x,y,z) = x^2y^3z^7 + \sin(z) + \sqrt{2}$

Corrigé

Solution Exercice I

$\frac{\partial f}{\partial x}(x,y) = 2x + y$

$\frac{\partial f}{\partial y}(x,y) = x + 4y^3$

$\frac{\partial f}{\partial x}(x,y) = e^y + 2xy$

$\frac{\partial f}{\partial y}(x,y) = xe^y + x^2$

$\frac{\partial f}{\partial x}(x,y) = 3x^2 - 3y$

$\frac{\partial f}{\partial y}(x,y) = 3y^2 - 3x$

Solution Exercice II

1- $\frac{\partial f}{\partial x}(x,y) = 6xy - y^3 - 1$

$\frac{\partial f}{\partial y}(x,y) = 3x^2 - 3xy^2 - 1$

$\frac{\partial^2 f}{\partial^2 x}(x,y) = 6y$

$\frac{\partial^2 f}{\partial^2 y}(x,y) = -6xy$

$\frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{\partial^2 f}{\partial y \partial x}(x,y) = 6x - 3y^2$ $\Longrightarrow$ *f(x,y) est une DTE*

2- $\frac{\partial f}{\partial x}(x,y) = \frac{x}{\sqrt{x^2+y^2}}$

$\frac{\partial f}{\partial y}(x,y) = \frac{y}{\sqrt{x^2+y^2}}$

$\frac{\partial^2 f}{\partial^2 x}(x,y) = \frac{y^2}{\sqrt{((x^2+y^2)^3}}$

$\frac{\partial^2 f}{\partial^2 y}(x,y) = \frac{x^2}{\sqrt{((x^2+y^2)^3}}$

$\frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{\partial^2 f}{\partial y \partial x}(x,y) = \frac{-xy}{\sqrt{((x^2+y^2)^3}}$ $\Longrightarrow$ *f(x,y) est une DTE*

3- $\frac{\partial f}{\partial x}(x,y) = \ln y + \frac{y}{x}$

$\frac{\partial f}{\partial y}(x,y) = \ln x + \frac{x}{y}$

$\frac{\partial^2 f}{\partial^2 x}(x,y) = \frac{-y}{x^2}$

$\frac{\partial^2 f}{\partial^2 y}(x,y) = \frac{-x}{y^2}$

$\frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{\partial^2 f}{\partial y \partial x}(x,y) = \frac{1}{y} + \frac{1}{x}$ $\Longrightarrow$ *f(x,y) est une DTE*

Solution Exercice III

1 − $\frac{\partial f}{\partial x}(x,y) = (4x + y + 7)\, f(x,y)$.

$\frac{\partial f}{\partial y}(x,y) = (x + 2y)\, f(x,y)$.

$\frac{\partial^2 f}{\partial^2 x}(x,y) = (4 + (4x + y + 7)^2).\, f(x,y)$

$\frac{\partial^2 f}{\partial^2 y}(x,y) = (2 + (x+2y)^2)\, f(x,y)$

$\frac{\partial^2 f}{\partial x \partial y}(x,y) = (1 + (x + 2y)(4x + y + 7))\, f(x,y)$.

$\frac{\partial^2 f}{\partial y \partial x}(x,y) = (1 + (x + 2y)(4x + y + 7))\, f(x,y)$

2- $f(x,y) = \sin(xy)$

$\frac{\partial f}{\partial x}(x,y) = y\cos(xy)$

$\frac{\partial f}{\partial y}(x,y) = x\cos(xy)$

$\frac{\partial^2 f}{\partial^2 x}(x,y) = -y^2 \sin(xy)$

$\frac{\partial^2 f}{\partial^2 y}(x,y) = -x^2 \sin(xy)$

$\frac{\partial^2 f}{\partial x \partial y}(x,y) = \cos(xy) - xy\sin(xy)$

3- $\frac{\partial f}{\partial x}(x,y) = \frac{1}{Ln10.(x+y)} = \frac{\partial f}{\partial y}(x,y)$

$\frac{\partial^2 f}{\partial^2 x}(x,y) = \frac{-1}{(x+y)^2} = \frac{\partial^2 f}{\partial^2 y}(x,y) = \frac{\partial^2 f}{\partial x \partial y}(x,y)$

Solution Exercice IV

1) $df = \frac{\partial f}{\partial x}(x,y)\, dx + \frac{\partial f}{\partial y}(x,y)\, dy$

$\frac{\partial f}{\partial x}(x,y) = \frac{2x+y}{y^2}$

$\frac{\partial f}{\partial y}(x,y) = -2\frac{x^2}{y^3} - \frac{x}{y^2}$

D'où $df = = \frac{2x+y}{y^2}\, dx + (-2\frac{x^2}{y^3} - \frac{x}{y^2})\, dy$

2) $\frac{\partial f}{\partial x}(x,y) = 2xy^3z^7 + 1$

$\frac{\partial f}{\partial y}(x,y) = 3x^2y^2z^7$

$\frac{\partial f}{\partial y}(x,y) = 7x^2y^3z^6 + \cos(z)$

$df = (2xy^3z^7 + 1)dx + (3x^2y^2z^7)dy + (7x^2y^3z^6 + \cos(z))\, dz$